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Critical state model on the activation energy for thermally activated flux avalanche in high-temperature superconductor

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Abstract Using the concept of critical state model and self-organized criticality of vortex motion, we have developed a model for $U(J)$, activation energy for thermally activated flux avalanche in high-temperature superconductor. A dynamic crossover between the flux avalanches and a pure thermally activated regime for $U(J)$ is investigated. The maximum value of the driving force responsible for thermal activation is shown to be $H/I \leq 0.665$. We also have the corresponding expressions for relaxation current $J(t)$ and characteristic relaxation time τ.

Keywords Magnetic relaxation, activation energy, avalanche effect, flux creep model, critical state model, self-organized criticality

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1. Introduction

Persistent supercurrents in high-T_c materials, decay [1] in time. The time decay of supercurrent or dissipation (due to transport current) is thought to be originated by the thermally activated motion of magnetic flux [2] inside the superconductor.

Theoretical idea of this phenomenon was first described by Anderson and Kim [3] (A-K), introducing the concept of thermal activation which is proportional to the rate $\exp\left[-U(J)/k_\beta T\right]$, where $U(J)$ is the activation energy, k_β, the Boltzmann constant and T, the absolute temperature. This process leads to a redistribution of flux lines or vortices (both flux and circulating currents) and hence of current loops associated with the magnetic moment which decay in time (so-called magnetic relaxation).

The standard flux creep model of Anderson and Kim assumes a linear dependence of the activation energy U on J given by [2-4]

$$U(J) = U_0(1 - J/J_c), \tag{1}$$

where J_c is the critical current density at which the barrier $U(J)$ vanishes [In fact, in the critical state [5], the barrier vanishes and thus the current density J is equal to J_c].

Larison and coworkers [6] pointed out that the $U(J)$ linear relationship is applicable only when $J - J_c << J_c$ and valid for the thermally activated relaxation processes. This linear approximation seems to be unsatisfactory for high-temperature superconductors. In general, the effective flux-pinning potentials [6,7] $U(J)$ should be a nonlinear function of current density J.

Zuning Wang and Donglu Shi [8] proposed a model of thermally activated flux avalanche dynamics for both high-T_c and conventional type-II superconductors using Maley method [9]. They showed that the flux motion is always thermally activated. But at large driving force (at large J), the flux motion is dominated by avalanche dynamics and at low driving force (at small J), the thermally activated flux avalanche effects are significantly reduced and the flux motion characteristics change to slow creep which is described by the A-K model. Thus, one can expect a dynamic crossover between the flux avalanches and a pure thermally activated regime. We have estimated the maximum value of the driving force responsible for thermal activation, using the concept of self-organized criticality (SOC) on the flux creep model.

2. *U-J* relationship for thermal avalanche

The activation energy U associated with the thermally activated vortex motion model assumes to be current dependent. To establish a relation for $U-J$, we use critical state model on flux dynamics.

Chao Tang [10] proposed an equation for flux motion velocity by taking into account both thermal activation and avalanche-like dynamics. He points out that flux motion is originally thermally activated, but the subsequent dynamics is governed by avalanches. The flux motion velocity is determined by two factors, namely, the flux hopping rate (in time scale) and the flux avalanche (in length scale). The flux motion velocity is written as

$$V = \omega s, \tag{2}$$

where the flux hopping rate is given by A-K model [4]

$$\omega = \omega_0 \exp. \frac{-U_0}{k_\beta T}(1 - J/J_c) \tag{3}$$

and the characteristic size for avalanche (SOC-like system[11]) as

$$s = s_c(1 - J/J_c)^{-\alpha}. \tag{4}$$

Therefore, the flux motion velocity for thermal avalanche is written as

$$V = \omega_0 s_c (1 - J/J_c)^{-\alpha} \exp. -\frac{U_0}{k_\beta T}(1 - J/J_c)^{\beta} \tag{5}$$

The parameters α and β are exponent constant related to each other by the relation (cf.Ref.[8]), $\alpha = \beta - 1$.

Assuming that the activation energy is originally thermally activated over the pinning barrier $U(J)$, the velocity of vortex motion (cf.Refs.[5,12]) for thermal avalanche (taking forward hopping only) can also be written as

$$V = V_0 \exp\left\lfloor -U_{eff}(J)/k_\beta T \right\rfloor, \tag{6}$$

where $U_{eff}(J)$ is the effective activation energy for thermally activated flux avalanches. Here, we identify $V_0 (= V_{max})$ is the maximum velocity of vortex motion for which $U_{eff}(J) = 0$ at $J = J_c$, the Bean critical state model [5]. This concept gives us effective activation energy for thermal-avalanche;

$$U_{th-avl} = U_0(1 - J/J_c)^{\beta} + \alpha k_\beta T \ln(1 - J/J_c) + k_\beta T \ln(V_0/\omega_0 s_c) \tag{7}$$

The first term represents the energy for thermal activation and second term represents the activation energy for avalanche effect. The last term represents the activation energy due to combined effect of thermally activated flux avalanches. Now, if we compare this with the result obtained by Wang and Shi[8], we have $V_0 = \omega_0 x_0$ where x_0 represent the average hopping distance when avalanches are considered.

The quantity defined by the eqs. (3 - 7) depends on J, the current density. According to the critical state model, $U_{eff}(J_c) = 0$ and on the basis of the forgoing equations, this occurs only when $\alpha \to 0$, $\beta \to 1$. This corresponds to the maximum velocity of the vortex motion for thermal avalanche given by

$$V_{max} = \omega_0 s_c . \tag{8}$$

Equating $V_{max} = V_0 = \omega_0 s_c$, the effective activation energy for thermal avalanche [eq.(7)] now becomes

$$U_{eff}(J) = U_0(1 - J/J_c)^{\beta} + k_\beta T \ln(1 - J/J_c)^{\alpha}. \tag{9}$$

The activation energy is now well separated. The first term represents the activation energy for thermal effects and the second term represents the activation energy for pure flux avalanches.

For small driving force, $J/J_c << 1$, eq. (9) can be approximated as

$$U_{eff}(J) \cong U_0(1 - \gamma J/J_c), \tag{10}$$

where

$$\gamma(T) = \beta + \alpha k_\beta T U_0^{-1}. \tag{11}$$

From eq.(11), it is clear that for $\alpha = 0.5$ and $\beta = 1.5$ (cf. Ref [8]), the maximum value of the driving force for thermal activation is $J/J_c \leq 0.665$ provided that $\alpha k_\beta T U_0^{-1} << 1$. The remaining part is responsible for avalanche effect which is experimentally unobservable [8].

3. Expressions for relaxation current $J(t)$

The basic equation governing the decay of current J can be written as [13-15]

$$dJ/dt = A \exp\left\lfloor -U_{eff}(J)/k_\beta T \right\rfloor \tag{12}$$

where A is a constant depending on the geometry of the superconducting samples.

To evaluate $J(t)$, we use eqs. (12) and (7) for thermally activated flux avalanche:

$$dJ/dt = A \exp\left[\ln(\omega_0 s_c/V_0) - \left\{\ln(1 - J/J_c)^{\alpha} + (U_0/k_\beta T)(1 - J/J_c)^{\beta}\right\}\right]. \tag{13}$$

To get the result obtained by Wang and Shi [8], we replace $\alpha = \beta - 1$ and integration of eq. (13) yields

$$\exp\left\{\left(U_0/k_\beta T\right)\left(1-J/J_c\right)^\beta\right\} = \left(-U_0 A\omega_0 s_c/J_c k_\beta T V_0\right)\beta t + C . \tag{14}$$

To evaluate the integration constant C, using the argument [see next section 4] at time $t = 0$ which corresponds to the critical state, $J(t) = J(0) \equiv J_c$ the unrelaxed current and eq. (14) implies $C = 1$. The final result for $J(t)$ is therefore:

$$J(t) = J_c - J_c\left[\left(k_\beta T/U_0\right)\ln\left(1+\beta t/\tau\right)\right]^{1/\beta}, \tag{15}$$

$$\tau = d^2 J_c k_\beta T/4U_0 H\omega_0 s_c,\ (V_0 = \omega_0 s_c). \tag{16}$$

Here, τ represents the characteristic relaxation time for thermal-avalanche.

For dynamic crossover, we use eqs.(12) and (10) for small driving force and the final result is

$$J(t) = J_c - J_c\left\{\left(\frac{k_\beta T}{\gamma U_0}\right)\ln\left(1+\gamma t/\tau^*\right)\right\}, \tag{17}$$

where $\gamma(T) = \beta + \alpha k_\beta T U_0^{-1}$ as before and the characteristic relaxation time is

$$\tau^* = \frac{d^2 J_c k_\beta T}{4U_0 H\omega_0 s_c}\exp\left[\frac{-U_0}{k_\beta T}(\gamma-1)\right],\ \gamma \ge 1 . \tag{18}$$

Using eqs.(1) and (12) and the proceeding as before, we

$$J(t) = J_c - J_c\left\{\left[\frac{k_\beta T}{U_0}\right]\ln(1+t/\tau')\right\}, \tag{19}$$

$$\tau' = d^2 J_c k_\beta T/4U_0 H\omega_0 s_c,\ (x_0 = s_c). \tag{20}$$

Here, τ' represents the characteristic relaxation time for pure thermal activation.

Discussion and conclusion

We have a phenomenological expression for the effective activation energy $U_{eff}(J)$ having two parts: (i) a pure thermal activation and (ii) avalanche-like dynamics. Our choice of the critical parameters $V_0 = V_{max} = \omega_0 s_c$, identify the part of $U_{eff}(J)$ responsible for thermally activated and avalanche-like dynamics. The maximum value of the driving force responsible for thermal activation is shown to be $J/J_c \le 0.665$. The relation (cf. Ref.[8]) $\alpha = \beta - 1$, clearly demonstrated here for the dynamic crossover between flux avalanches and a pure thermal activation. For pure thermal activation, $\alpha \to 0$, $\beta \to 1$, and $U_{eff}(J)$ are linear as expected in the A-K model [3]. Similar argument holds for the relaxation current $J(t)$.

We have calculated the relaxation current $J(t)$ for the corresponding $U(J)$. Our procedure for the time evaluation of flux distribution or current relaxation is based on the framework of Bean critical state model [5]. The basic assumption of the critical state model is that a superconductor is capable of sustaining 'virtually' loss-less currents up to a critical current density J_c, but not beyond this. So, the loss-less current means unreleased current and we can define it at the initial time $t = 0$, $J(t) = J_c$. If the current flowing throughout the entire sample is J_c, this is said to be in a 'critical state'. As the flux rushes in, the flux profile relaxes and the current density is no longer critical but decays as time progresses.

At time $t = 0$, upon application of a static applied field H, a flux profile is set up consistent with static equilibrium, which implies $U_{eff}(J) = 0$ and indicates that there exists no barrier for flux motion. So at the critical state, $t = 0$, $J(t) = J_c$ and $U_{eff}(J) = 0$. This criterion has been used to evaluate the integration constant C [see eq. (14)] in deriving the eqs. (15), (17) and (19).

We have obtained the characteristic relaxation times τ', τ and τ^* for a superconducting slab sample of thickness 'd' in a competitive way (see Refs. [8,13-15]). The new parameter γ related to the exponent constants α and β, depends on temperature. At very low temperature, $\gamma \to \beta$ and becomes unity for pure thermal activation. The parameter τ^* can be approximated numerically. For the sample Bi 2:2:1:2 reported by Wang and Shi [8], taking $U_0 = 0.027eV$, $\alpha = 0.5$, $\beta = 1.5$ and $k_\beta T = 1.032\times10^{-5}eV$; for $T = 12$K, we have $\gamma = 1.519$. The characteristic relaxation time τ^* is found to be 1.26×10^{-4} s, for $\tau = 100$ s.

In conclusion, our proposed model of U(J) is consistent with the result obtained by Wang and Shi [8]. We confirm the relation $\alpha = \beta - 1$, for dynamic crossover and estimated the maximum value of the driving force responsible for thermal activation. Evaluation of $J(t)$, τ and τ^* ($<< \tau$) are consistent with other observations [13-15].

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